

Performances of Spline-Laplacian Filters in a Two-Class EEG Experiment

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Abstract

The spherical and the New Orleans Laplacians were compared in the classification of a two-class experiment involving a stimulus driven condition and two variants of imagining the stimulus. The spherical filter produced the best results for all conditions and was the more accurate one in a test with simulations.

Index Terms

High-resolution EEG; Surface Laplacian; Single-trial classification; Two-class EEG problem.

I. INTRODUCTION

THE applications of Surface Laplacian (SL) filtering in EEG are mainly supported by studies that analyze the accuracy of filters through simulated potentials. Data generated from layer-head models are commonly employed, and parameters like noise and conductivity are controlled [1], [2], [3]. Although simulations are a convenient way of assessing methods, real experiments are necessary to complete such evaluations. However few studies have been specifically designed to evaluate SL filters under real experimental conditions. With the intention of helping to close this gap, we used real data to test two popular SL filters in EEG single-trial classification. We compared the performances of the New Orleans [2] SL (NOSL) and the Spherical SL (SSL) [4], [5] in multi-channel,

single trial classifications of EEG data extracted from experiments on stimulus-driven and imagined-images. In both SL approaches the potential at scalp position $\mathbf{r}$ at a given instant is described by [2], [4], [5]

$$V(\mathbf{r}) = \sum_{i=1}^N c_i K(\mathbf{r}, \mathbf{r}_i) + \sum_{i=1}^M d_i \phi_i(\mathbf{r}), \quad (1)$$

where the coefficients c_i and d_i depend on the set of measured potentials, labeled $V_i, i = 1, \dots, N$, and $\mathbf{r}_i$ is the location of the i th-electrode. In the NOSL approach $K(\mathbf{r}, \mathbf{r}_i) = |\mathbf{r} - \mathbf{r}_i|^4 \log(|\mathbf{r} - \mathbf{r}_i|^2 + w^2)$, where $w \in [0, 1]$ is a free parameter, and $M = 10$ with the functions $\phi_i, \dots, \phi_{10}$ corresponding to the monomials $1, x, y, x^2, xy, y^2, z, xz, yz, z^2$, respectively. For the SSL, $\phi = 1$, thereby $M = 1$, and $K(\mathbf{r}, \mathbf{r}_i)$ corresponds to the harmonic function $g(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}_i) = (1/4\pi) \sum_{n=1}^{\infty} [(2n+1)/n^m(n+1)^m] P_n(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}_i)$, where carets denote unit vectors, m is the interpolation order, and P_n is the Legendre polynomial of degree n . In our implementation g was calculated by keeping Legendre polynomials up to degree 50. The coefficients c_i and d_i are determined from [2], [4], [5]

$$(\mathbf{K} + \lambda \mathbf{I}) \mathbf{c} + \mathbf{T} \mathbf{d} = \mathbf{v}, \quad (2a)$$

$$\mathbf{T}^T \mathbf{c} = 0, \quad (2b)$$

where $\mathbf{v}, \mathbf{c} \in \mathbb{R}^N$ and $\mathbf{d} \in \mathbb{R}^M$ are vectors of components V_i, c_i and d_i , respectively, and $\mathbf{K} \in \mathbb{R}^{N \times N}$ and $\mathbf{T} \in \mathbb{R}^{N \times M}$ are matrices of elements $K(\mathbf{r}_i, \mathbf{r}_j)$ and $\phi_k(\mathbf{r}_j)$, respectively. The parameter λ applies only to the SSL (i.e. $\lambda = 0$ for the NOSL) where it regulates the trade-off between minimizing the square error of the potentials' fitting and the interpolation's smoothness. The solution of (2) is obtained algebraically: $\mathbf{c} = \mathbf{C} \mathbf{v}$ and $\mathbf{d} = \mathbf{D} \mathbf{v}$, where $\mathbf{C} = (\mathbf{K} + \lambda \mathbf{I})^{-1} (\mathbf{I} - \mathbf{T} \mathbf{D})$ and $\mathbf{D} = [\mathbf{T}^T (\mathbf{K} + \lambda \mathbf{I})^{-1} \mathbf{T}]^{-1} \mathbf{T}^T (\mathbf{K} + \lambda \mathbf{I})^{-1}$. The SL waveforms are obtained by applying the SL operator ∇_{surf}^2 to (1). At the electrode sites they can be computed by linearly transforming the EEG potentials. To see this, let $\Phi = (\phi_{ij})$ be a N -by- S spatiotemporal matrix containing the EEG data, where the rows correspond to the measurement electrodes and the columns to the time samples. Then, the corresponding SL data set is a matrix of the same size as Φ , given by

$$\mathbf{Y} = \mathbf{L} \Phi, \quad (3)$$

where $\mathbf{L} = \tilde{\mathbf{K}} \mathbf{C} + \tilde{\mathbf{T}} \mathbf{D}$ and $\tilde{\mathbf{K}}$ and $\tilde{\mathbf{T}}$ are matrices which (i, j) -entries are $\nabla_{\text{surf}}^2 K(\mathbf{r}_i, \mathbf{r}_j)$ and $\nabla_{\text{surf}}^2 \phi_i(\mathbf{r}_j)$, respectively. The advantage of using Eq. (3) in computing the SL is that $\mathbf{L}$ depends only on the electrode locations. Therefore it can be computed once, provided the electrode setup remains the same.

The SSL is derived from the spline theory developed by Wahba [6], [7], [8], where Eq. (2) determines c_i and d_i in order to solve a variational problem in a (Hilbert) space of functions spanned by K and $\phi_1, \dots, \phi_M$. In this framework, K is an unique reproducing kernel spanning a N -dimensional subspace where the constraints $V(\mathbf{r}_i) = V_i$ are satisfied. The functions ϕ_i are unconstrained and Eq. (2b) is derived from an orthogonal condition. As a result

the so-called spline smoother provides the best mean squared error fit to the interpolated data. The NOSL does not have a similar mathematical background. Although it is presented in [2] as the 3rd-order thin-plate spline in 3D, the NOSL function K actually corresponds to a modification of the kernel used by the 2D thin-plate spline of 3rd-order, which is given by $K(\mathbf{r}, \mathbf{r}_i) = |\mathbf{r} - \mathbf{r}_i|^4 \log |\mathbf{r} - \mathbf{r}_i|$ where $\mathbf{r}$ and $\mathbf{r}_i$ are vectors in $\mathbb{R}^2$. The unique kernel for the 3D thin-plate spline does not involve the logarithm function but reads [8] $K(\mathbf{r}, \mathbf{r}_i) = |\mathbf{r} - \mathbf{r}_i|^{2m-3}$, where $m > 2$ defines the order of the interpolation. The NOSL has also the drawback that the matrix $\mathbf{K}$ in (2) is ill-conditioned for some electrode setups. To handle this problem, [2, p. 585] suggested the replacement of electrode coordinates that are exactly zero by a small number. For this reason, all occurrences of zero in our electrode coordinates were replaced by 0.001.

II. METHODS

Experimental Procedure

Eleven healthy subjects (labeled S1–S11) participated in the two-class experiment conducted in the CSLI Suppes Brain Lab. Each subject was randomly presented on every other trial either a visual stimulus, consisting of a “stop” sign flashed on a computer screen for 300 ms, or an auditory stimulus consisting of the English word “go” presented via computer speaker (digitized speech at 22 kHz lasting 300 ms). Subjects S1–S7 were asked to form a mental image of the stimulus experienced on the previous trial, whereas subjects S8–S11 were instructed to imagine the alternative stimulus. Thus, three experimental conditions were generated, one stimulus-driven and two variants of imagining. They were labeled STIM, IMAG1, and IMAG2, respectively. A total of 600 trials of 1000 ms duration, of which we analyzed the first 500 ms, were recorded for each subject and each condition. The EEG data were recorded from 64 scalp locations of the 5% system of Ref. [9], with the exceptions of the electrodes Nz, AF1, AF2, AF5, AF6, T9, T10, P9, P5, P6, P9, P10 and those labeled PO and I. Two other electrodes were placed below the eyes corresponding to HEOG and VEOG channels. The analog signal was amplified and band-pass filtered with an analog filter of 0.1 to 300 Hz and a 60 Hz notch filter and recorded at a sample rate of 1 kHz. The data were downsampled for analysis by a ratio of 16:1, cutting down the bandwidth from 500 Hz to 31.25 Hz. No artifacts were removed or rejected.

Feature Extraction, SL Parameter Selection, and Classification

Multi-channel, single-trial classifications were independently performed over either the unfiltered matrix Φ and the SL-filtered matrices $\mathbf{Y}_{\text{SSL}}$ and $\mathbf{Y}_{\text{NOSL}}$ obtained from (3) by using SSL and NOSL. The classifications were performed for each one of the subjects and experimental conditions. The trials of the input data were classified in two classes, “stop” or “go,” using the linear discriminant algorithm of Ref. [10] with 10-fold cross validation (CV), to

ensure the statistical significance of the results. Feature reduction was performed using single-value decomposition (SVD), followed by a classical linear-discriminant classification. The parameters to be optimized were the number k of selected principal components (in the 1-200 range), along with the NOSL parameter w or the SSL pair (m, λ) , over regular grids covering the common ranges in the literature, namely $w = 0, \dots, 1$, in steps of 0.1, $m = 2, \dots, 5$ and $\lambda = 10^{-4}, 10^{-5}, 10^{-6}$. The cross validation consisted of a main loop, where the 600 trials were partitioned into 10 sets of 60 trials. For each CV step of the loop, one of the sets was taken as the test set and the union of the others formed the training set. In the nested loop, the training set was subpartitioned in the same fashion into 10 pairs of subtraining and subtest sets. Using these sub sets, ten classifications were performed per value of k and per set of SL parameters. In total, we obtained 2000 rates (10 rates $\times$ 200 k -values) for the unfiltered data, 22000 rates (10 rates $\times$ 200 k -values $\times$ 11 w -values) for the NOSL, and 24000 rates (10 rates $\times$ 200 k -values $\times$ 12 (m, λ) -pairs) for the SSL. The mean over each set of 10 rates was computed and the maximum mean value defined the best k as well as the best set of SL parameter. The test set of the outermost CV was then classified using these optimized values of the parameters. The entire procedure was repeated for each of the 10 outermost folds, providing 10 classification rates for each input matrix. The 10 rates were averaged to yield the mean classification rates presented in Table I. The significance of the means was estimated by computing the p -values under the null hypothesis of random labeling of the trials, as in [10], [11], [12].

III. RESULTS

Table I summarizes the results of the classification. Each entry corresponds to a mean over the 10 rates coming from the 10-fold validation. All rates are above chance. With few exceptions, the NOSL filter had the worst performance. It improved the rates of the unfiltered data for only 4 data sets (S7/STIM, S10/STIM, S6/IMAG1, and S10/IMAG2) and outperformed the SSL filter in just one case (S6/IMAG1). On the other hand, the spherical-filtered data provided the best rates for 18 of the 22 data sets. A supplementary Table showing the occurrences of each tested SL parameter as the optimal one in the classification is available online. Despite [2] recommendation to use w within the interval 0.5–1.0, $w < 0.5$ occurred more often than larger ones. For the spherical Laplacian, the 4th-order interpolation ($m = 4$) with $\lambda = 10^{-5}$ gave the best results for the three experimental conditions.

The significance of the differences in performance observed in Table I was evaluated and the results are showed in Table II. Because of the asymmetry of our rate distributions, two non-parametric tests were used instead of the classic ANOVA. Namely, paired permutation tests [13], [14] and the sign test. The first three columns of numerical data refer to the outcomes of the paired permutation tests and the last one to the result of the sign test. The paired permutation tests [13] were computed using the classification rates obtained before averaging in the 10-fold cross-validation. Thus, 110 rates were used to perform the comparisons of the data sets STIM, 70 for IMAG1, and 40

Table I

MEAN CLASSIFICATION RATES OBTAINED BY AVERAGING OVER THE 10 SETS OF TEST TRIALS WITH 60 TRIALS IN EACH TEST. ALL p -VALUES ARE LESS THAN 10^{-11} FOR THE CONDITION STIM AND ARE NOT SHOWED. THE CONDITIONS IMAG1 (S1–S7) AND IMAG2 (S8–S11) WERE CONCATENATED UNDER THE LABEL IMAG AND THEIR p -VALUES ARE DISPLAYED IN THE RIGHT COLUMN.

Subject	Unfiltered data			NOSL -filtered data			SSL-filter data		
	STIM	IMAG		STIM	IMAG		STIM	IMAG	
S1	99.5	80.2	$< 10^{-4}$	95.3	74.8	0.0001	99.3	84.0	$< 10^{-4}$
S2	97.8	62.3	0.0259	92.8	56.5	0.1831	99.7	63.2	0.0259
S3	97.8	67.0	0.0031	94.8	59.5	0.0775	99.2	70.0	0.0005
S4	99.7	66.8	0.0031	97.3	52.7	0.3494	99.5	67.0	0.0031
S5	96.3	68.3	0.0013	90.7	60.0	0.0462	97.5	69.3	0.0013
S6	95.2	55.7	0.1831	93.3	56.3	0.1831	96.7	55.5	0.1831
S7	97.3	65.8	0.0067	98.7	63.0	0.0259	98.8	69.7	0.0013
S8	97.2	75.8	$< 10^{-4}$	96.2	71.5	0.0005	98.2	75.0	$< 10^{-4}$
S9	97.8	58.8	0.0775	97.0	52.3	0.3494	99.3	60.5	0.0462
S10	98.7	75.5	$< 10^{-4}$	99.2	78.3	$< 10^{-4}$	99.3	78.7	$< 10^{-4}$
S11	97.8	67.0	0.0031	97.0	64.0	0.0137	99.0	67.2	0.0031

Table II

RESULTS (p -VALUES) OF PAIRED PERMUTATION TESTS OF THE NULL HYPOTHESIS THAT METHOD X AND METHOD Y ARE EQUAL. THE COMPUTATION WAS PROCESSED OVER THE NON-AVERAGED CLASSIFICATION RATES BY TAKING 10^5 PERMUTATION SAMPLES [13]. THE LAST COLUMN SHOWS THE OUTCOMES OF THE SIGN TEST PERFORMED OVER THE 22 MEANS.

X vs. Y	permutation tests			sign test
	STIM	IMAG1	IMAG2	
unfiltered vs. NOSL	$< 10^{-5}$	$< 10^{-5}$	0.03	0.0009
SSL vs. unfiltered	10^{-5}	0.05	0.39	0.004
SSL vs. NOSL	$< 10^{-5}$	$< 10^{-5}$	0.006	10^{-5}

for IMAG2.

Table II confirms the comparative lower performance of the NOSL filter. The permutation tests showed this filter was outperformed by the other methods in STIM and IMAG1 conditions at high significance level. A significant difference is also observed in IMAG2 in favor of the SSL filter against the NOSL filter. For the particular case of IMAG2 we cannot reject the null hypothesis that any difference between unfiltered/NOSL and SSL/unfiltered are not significant. However the SSL filter improved the results of the unfiltered data for conditions STIM and IMAG1 at good level.

The outcomes of the pairwise sign-test analysis are shown in the last column of Table II. The Twenty-two rates per method of Table I were used in the comparison. Overall the results reinforce the outcomes of the permutation tests. The low performance of the NOSL filter relative to the other methods is observed again at high significance level. Now the spherical filter outperforms the unfiltered data with $p = 0.004$ significance.

Simulations were used to test the accuracy of the two SL methods in matching the analytic Laplacian. The simulations involved sets of 1-20 radial dipoles on a 4-sphere model, with brain, CSF, skull, and scalp shells set with radii 8, 8.1, 8.6, and 9.2 cm, and the conductivity ratios as 0.2 (brain/CSF), 40 (brain/skull), and 1 (brain/scalp).

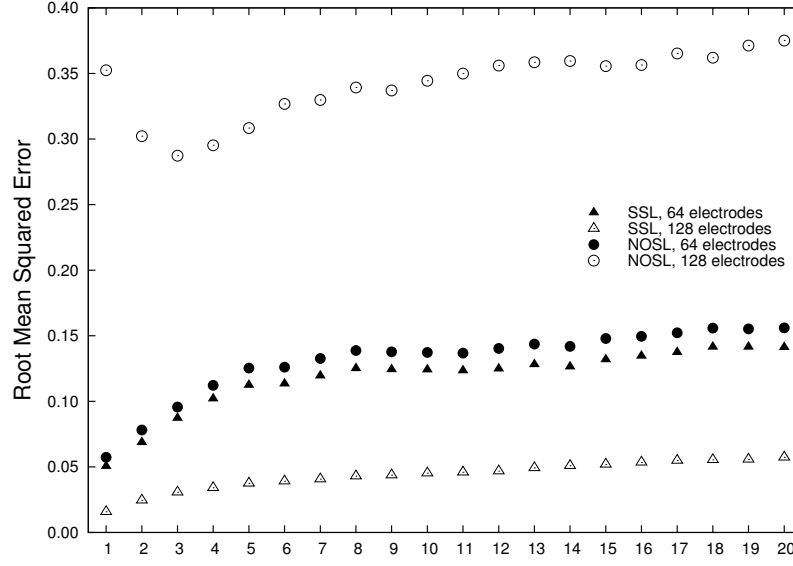


Figure 1. Simulated RMS errors of the Laplacian estimators as a function of the number of dipoles. Sources in the simulation were 1-20 radial dipoles randomly orientated and located inside the brain. RMS errors of the estimations were with respect to the analytic Laplacian computed as a function of the number of sources. Each point in the graph corresponds to the mean of 100 random configurations. The SSL was computed with $m = 4$ and without λ correction. The NOSL was computed with $w = 1$.

One hundred configurations were generated for each set of dipoles. In each configuration, the dipoles orientation and locations were randomized inside the brain shell with a minimum distance of 5 cm from the center of the spheres. The generated scalp potentials were sampled with two montages. The first montage was the one used in the experiments, and the other corresponded to the 128-channel HydroCel GSN system. The SSL and NOSL algorithms were applied to estimate the Laplacian from the simulated scalp-potential distributions. The root mean square error (RMSE) of the Laplacian estimations with respect to the analytic SL was computed, and the obtained errors were averaged across all configurations of same number of dipoles. The results are shown in Figure 1.

IV. DISCUSSIONS AND CONCLUSIONS

We compared the performance of the NOSL and SSL filters in the multi-channel, single-trial classification of a two-class EEG experiment with a 64-channel-EEG montage. The parameter spaces of the filters were partially explored to seek optimal implementations. The classification rates achieved were used to compare the filters against each other and also to compare them to the unfiltered data. Two nonparametric tests were used to make the statistical comparisons. Both tests showed that the NOSL filter was outperformed by the SSL filter with high significance levels. The NOSL filter also failed to improve the rates of the unfiltered data. On the other hand, a significant improvement relative to the rates of the unfiltered data was achieved in the stimulus-driven data sets by the SSL. The permutation test also showed a difference in favor of the SSL filter for the condition IMAG1 at level $p = 0.05$, but no significant overall difference was observed for the data sets of IMAG2. In general terms, the outcomes of the sign test showed close agreement with the permutation test, favoring SSL over the NOSL filter. Additionally,

simulated potential distributions over a 4-sphere model were used to assess the accuracy of the estimators. The data was generated by configurations of 1-20 sources and sampled with setups of 64 and 128 electrodes. RMS errors of SL estimations in respect to the analytic solution showed the SSL more accurate than the NOSL regardless source configuration. The SSL error diminished with increasing the number of channels, while the NOSL showed to be more accurate for the 64-electrode setup. This difference in accuracy offers a plausible explanation for the results achieved with the real experiments and requires consideration, especially when high spatial samplings are used in the estimations.

Acknowledgment. We would like to thank Logan Groseknick for helpful discussions and suggestions.

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