

Learning Pattern Recognition through Quasi-synchronization of Phase Oscillators

Ekaterina Vassilieva, Guillaume Pinto, J. Acacio de Barros, and Patrick Suppes

Abstract—The idea that synchronized oscillations are important in cognitive tasks is receiving significant attention. In this view, single neurons are no longer elementary computational units. Rather, coherent oscillating groups of neurons are seen as nodes of networks performing cognitive tasks. From this assumption, we develop a model of stimulus-pattern learning and recognition. The three most salient features of our model are: a new definition of synchronization, demonstrated robustness in the presence of noise, and pattern learning.

Index Terms—Kuramoto oscillators; pattern recognition; phase oscillators; oscillator network; quasi-synchronization.

I. INTRODUCTION

Oscillator synchronization is a common phenomenon. Examples are the synchronizations of pace-maker cells in the heart [43], of fireflies [43], of pendulum clocks [3], and of chemical oscillations [17]. Winfree introduced and formalized the concept of biological oscillators and their synchronization [43]. Later, Kuramoto [17] developed a solvable theory for this kind of behavior.

To understand how oscillators synchronize, let us consider neural networks. Let A be a neuron that fires periodically. A is our oscillator, with natural frequency given by its firing rate. Now, if another neuron B , coupled to A , fires shortly before A is expected to fire, this will cause A to fire a little earlier than if B did not fire. If you have many neurons coupled to A , each neuron will pull A 's firing closer to its own. This is the overall idea of Kuramoto's model [17]. In it, a phase function encodes neuron firings. The dynamics of this phase is such that it is pulled toward the phase of other neurons. It can

be shown that if the couplings are strong enough, the neurons synchronize (for a review, see [1]).

A question of current interest is the role of neural oscillations on cognitive functions. In theoretical studies, synchronous oscillations emerge from weakly interacting neurons close to a bifurcation [9], [13]. Experimentally, Gray and collaborators [10] showed that groups of neurons oscillate. Neural oscillators are apparently ubiquitous in the brain, and their oscillations are macroscopically observable in electroencephalograms [9]. Experiments not only show synchronization of oscillators in the brain [7], [8], [16], [19], [22], [26], [27], [28], [38], [29], [40], but also their relationship to perceptual processing [8], [16], [18], [22], [28]. Oscillators may also play a role in solving the binding problem [7], and have been used to model a range of brain functions, such as pyramidal cells [20], electric field effects in epilepsy [25], cat's visual cortex activities [28], birdsong learning [39], and coordinated finger tapping [44]. However, current techniques for measuring synchronized neuronal activity in the brain are not good enough to unquestionably link oscillatory behavior to the underlying processing of cognitive tasks.

During the past fifteen years, researchers tried to build oscillator and pattern recognition models inspired by biological data. As a result, diverse computational models based on networks of oscillators were proposed. Ozawa and collaborators produced a pattern recognition model able to learn multiple multiclass classifications online [24]. Meir and Baldi [2] were among the first to apply oscillator networks to texture discrimination. Wang did extensive work on oscillator networks, in particular with LEGION networks [42], employing oscillator synchronization to code pixel binding. Wang and Cesmeli computed texture segmentation using pairwise coupled Van Der Pol oscillators [41]. Chen and Wang showed that locally coupled oscillator networks could be effective in image segmentation [6]. Borisjuk and collaborators studied a model of a network of peripheral oscillators controlled by a central one, and applied it to problems such as object selection [15] and novelty detection [4].

In this paper we apply networks of weakly-coupled Kuramoto oscillators to pattern recognition, an important topic in . Our main goal is to use oscillators in

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a way that allows learning. To allow for a richness of synchronization patterns, and therefore prevent the systemic synchronization of oscillators, we work with weaker couplings than what is required for robust synchronization [1]. Such couplings require us to depart from the standard definition of synchronization, leading us to redefine synchronization in a weaker sense. This paper is organized as follows. Section II motivates our definition of quasi-synchrony in pattern recognition. Section III show how learning can occur by changes to their frequencies. Section IV applies the oscillator model to image recognition. Finally, we end with some comments.

II. PATTERN RECOGNITION WITH WEAKLY COUPLED OSCILLATORS

We start with a set of N weakly-coupled oscillators, $O_1, \dots, O_N$, and split this set in two: stimulus and recognition [32]. Formally, $G = \{O_1, O_2, O_3, \dots, O_N\}$ is the network of oscillators, and G_S and G_R are the stimulus and recognition subnetworks of G , such that $G = G_S \sqcup G_R$. For our purposes, the stimulus subnetwork represents neural excitations due to an external sensory signal, and synchronization pattern in the recognition subnetwork represents the brain's representation of the recognized stimulus. We assume that synchronizations of oscillators represent information processed in the brain. Each oscillator O_n in the network is characterized by its natural frequency f_n . The couplings between oscillators is given by a set of non-negative coupling constants, $\{k_{nm}\}_{m \neq n}$. For simplicity, we assume symmetry, i.e. $k_{nm} = k_{mn}$ for all n and m .

Let us assume that we can represent O_n by a measurable quantity $x_n(t)$. If we write $x_n(t)$ as $x_n(t) = A_n(t) \cos \phi_n(t)$, then $\phi_n(t)$ is the phase and $A_n(t)$ the amplitude. Assuming constant amplitudes, we focus on phases satisfying Kuramoto's equation [17]

$$\frac{1}{2\pi} \frac{d\phi_n}{dt}(t) = f_n + \sum_{m=1}^N A_n A_m k_{nm} \sin[\phi_m(t) - \phi_n(t)]. \quad (1)$$

We define a stimulus s as a set of ordered triples

$$s = \{(A_n^s, f_n^s, \phi_n^s(0))\}_{n \in G_S}, \quad (2)$$

with each triple representing the amplitudes, natural frequencies, and initial phases of an oscillator. Intuitively, s is meant to be a model of the brain's sensory representation of an external stimulus. When a stimulus is presented, the phases of the stimulus oscillators, as well as their natural frequencies and amplitude, match the

values in s . In other words, for all oscillators $O_n \in G_S$, when s is presented, $f_n = f_n^s$, $A_n = A_n^s$, $\phi_n(0) = \phi_n^s(0)$.

A typical phenomenon in a network of Kuramoto oscillators is the emergence of synchronization. Two oscillators are considered synchronized if they oscillate with the same frequency and are phase locked [12], [14]. Let us consider a six-oscillator example, with two stimulus, O_1 and O_2 , and four recognition, O_3 , O_4 , O_5 , and O_6 , oscillators having couplings $k_{mn} = 1$, except $k_{12} = k_{21} = 0$. We set

$$f_3 = 10 \text{ Hz}, \quad f_4 = 15 \text{ Hz}, \quad (3)$$

$$f_5 = 20 \text{ Hz}, \quad f_6 = 25 \text{ Hz}, \quad (4)$$

as the natural frequencies of the recognition oscillators. Since (1) implies varying frequencies, we define the instantaneous frequency of the i -th oscillator as the temporal rate of change of its phase, i.e., $\omega_i = d\phi_i/dt$. At this point we must make our notation explicit. Both f_i and ω_i are frequencies, but f_i enters in (1) as the natural frequency of an oscillator, and is measured in cycles per second, whereas ω_i is defined as the time derivative of ϕ_i , and is measured in radians per second. We emphasize that these two frequencies are not only measured differently, but they are also conceptually distinct. Usually, there is no need to make such distinction, but we will need it later on when we discuss learning. Figs. 1–3 shows the instantaneous frequency of the oscillators for three different stimuli (the natural frequencies are shown as straight lines, for reference).

We can quantify the synchronization (or lack of) in Figs. 1 and 2. In Fig. 3 the situation is different. There,

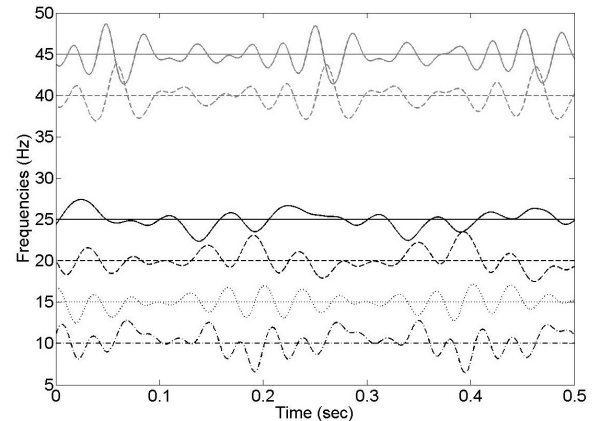


Figure 1. Six-oscillator network response to stimulus $f_1^s = 40$ Hz, $f_2^s = 45$ Hz, $A_1^s = 1$, $A_2^s = 1$, $\phi_1^s(0) = 0$, and $\phi_2^s(0) = 0$. Oscillators do not synchronize. O_1 and O_2 are the dashed and solid grey lines, and O_3 , O_4 , O_5 , and O_6 are the dash-dot, dotted, dashed, and solid black lines.

two groups seem to emerge, with frequencies varying

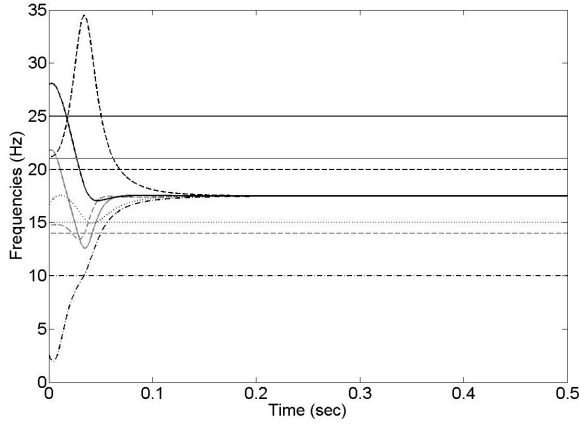


Figure 2. Six-oscillator network response to stimulus $f_1^s = 14$ Hz, $f_2^s = 21$ Hz, $A_1^s = 4$, $A_2^s = 4$, $\phi_1^s(0) = 0$, and $\phi_2^s(0) = 0$. Oscillators synchronize completely after approximately 150 ms. O_1 and O_2 are the dashed and solid grey lines, and O_3 , O_4 , O_5 , and O_6 are the dash-dot, dotted, dashed, and solid black lines.

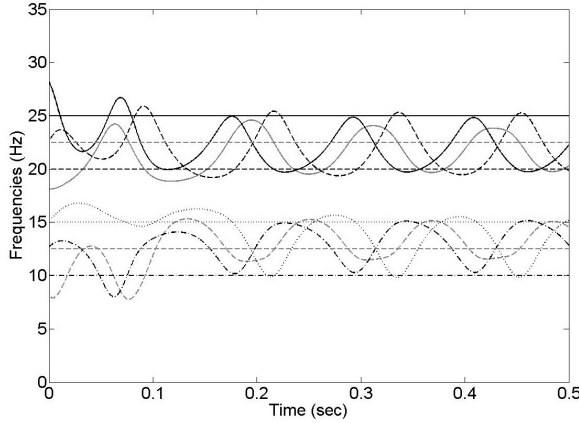


Figure 3. Six-oscillator network response to stimulus $f_1^s = 12.5$ Hz, $f_2^s = 22.5$ Hz, $A_1^s = 2$, $A_2^s = 2$, $\phi_1^s(0) = 0$, and $\phi_2^s(0) = 0$. O_1 and O_2 are the dashed and solid grey lines, and O_3 , O_4 , O_5 , and O_6 are the dash-dot, dotted, dashed, and solid black lines. Oscillators' behavior display varying instantaneous frequencies that seem to show that the group of oscillators O_1 , O_3 , and O_4 oscillate coherently, as well as group O_2 , O_5 , and O_6 .

periodically within each group. The standard definition states that two oscillators O_n and O_m are synchronized if their frequencies are asymptotically the same, i.e. if

$$\lim_{t \rightarrow +\infty} \left[\frac{d\phi_n}{dt}(t) - \frac{d\phi_m}{dt}(t) \right] = 0. \quad (5)$$

This definition works nicely for Figs. 1 and 2, but fails for Fig. 3. If we want to say that the oscillators in Fig. 3 are synchronized, we need to propose a different definition. For instance, the periodic variations in the differences between the frequencies of O_3 and O_4 result from various perturbations induced by the other oscillators, to which they are connected but not synchronized. To address this point, Kazanovich and

Borisyyuk [15] proposed that two oscillators are synchronized if their dephasing, i.e., the difference between their phases, is bounded. This definition is not adequate for our purposes, since for a finite time all continuous functions are bounded, and we may not differentiate non-synchronized oscillators with close natural frequencies from synchronized oscillators undergoing substantial perturbations (see Fig. 3). Therefore, we need a more flexible definition of synchronization.

Let Φ and Ψ be two continuous random variables independently and uniformly distributed on the interval $[0, 2\pi]$. Then $\sin \Phi$ and $\sin \Psi$ have zero expectation, and $\text{Var}(\sin(\Phi - \Psi)) = 0.5$. However, if Φ and Ψ are perfectly correlated, then $\text{Var}(\sin(\Phi - \Psi)) = 0$. For the example shown in Fig. 3, we illustrate in Fig. 4 the variance of the sine of the phase differences (dephasing). We see that for O_3 and O_4 the sine of the dephasing is constrained to a small interval, causing its variance to be small. On the other hand, the sine of the dephasing between O_3 and O_5 , which are intuitively not synchronized, looks like a sine function, and its variance is approximately 0.5. So, we adopt the following.

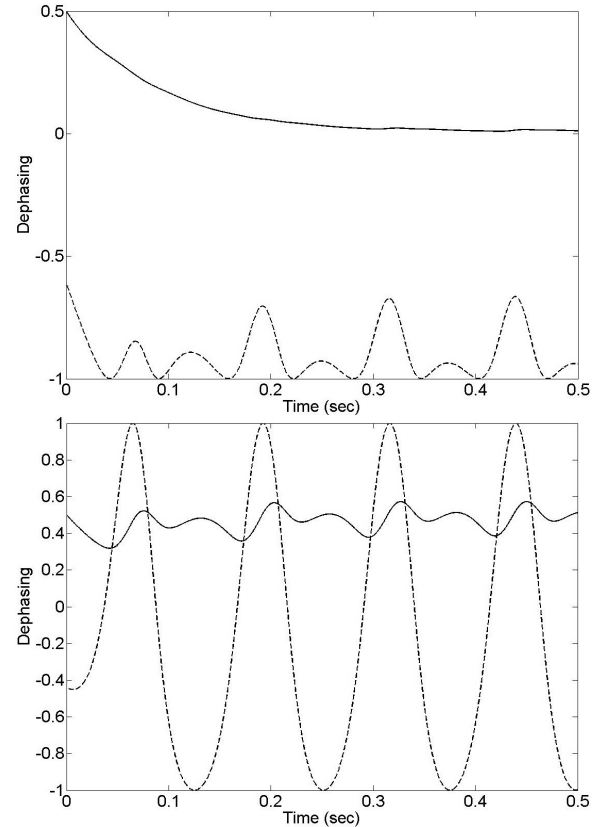


Figure 4. Sines of the phase differences and their variances for oscillator pairs O_3 and O_4 (top) and pairs O_3 and O_5 (bottom). The sines are given by dashed lines, and the solid lines show recursive numerical estimations of the variance.

Definition 1: Oscillators O_n and O_m are ϵ -quasi-

synchronized (or *quasi-synchronized*) if their phases, represented by Φ_n and Φ_m , satisfy

$$\text{Var}[\sin(\Phi_n - \Phi_m)] < \epsilon, \quad 0 < \epsilon \ll 0.5. \quad (6)$$

The smaller the value of ϵ , the closer quasi-synchronization is to be equivalent to (5). But it is possible for oscillators to be quasi-synchronized and to not satisfy (5). In fact, in Section IV the example only works if we consider quasi-synchronization. Even though ideally ϵ should be as close to zero as possible, throughout this paper we use $\epsilon = 0.35$. This value was chosen because, in our simulations, it allows for a quicker recognition of synchronization (due to its high value) without loss of discrimination between patterns. In other words, for our finite-time simulations, very small values of ϵ would take too long to converge, whereas values closer to 0.5 would not discriminate unsynchronized oscillators.

To further investigate the differences between quasi- and standard synchronization, it is useful to see how our example behaves when we vary the stimulus. First, let us recall that, in the mean-field approximation, with the assumption of equal weight and all-to-all couplings, the oscillators synchronize when the mean coupling exceeds a critical value $K_c = 2/(\pi g(\omega_0))$, where $g(\omega)$ is the density distribution of oscillator frequencies and ω_0 its mean (g is assumed symmetric) [17]. Our example violates those assumptions, mainly the all-to-all equal coupling, the symmetric distribution of frequencies, and the large number of oscillators. But, as Fig. 1 shows, if we pick O_1 and O_2 close to O_3, O_4, O_5 , and O_6 in a more symmetric way, all oscillators synchronize. On the other hand, if O_1 and O_2 are far from symmetry, the oscillators do not synchronize (Fig. 2). More interestingly, there are regimes of quasi-synchronization for other frequency distributions. To make this explicit, let us look at Fig. 5, which shows the synchronization patterns for stimulus frequencies varying from 5 Hz to 45 Hz. The results of Fig. 5 are fairly general, as long as we do not vary the couplings too much, as very strong coupling would yield systematic synchronization whereas very weak coupling would yield no synchronization. But the different areas would only be less smooth if we were to include noise. We see that, given our couplings, synchronization happens when both stimulus oscillators are around 17.5 Hz, the mean frequency of O_3, O_4, O_5 , and O_6 . For this case, all oscillators in the network synchronize, and this is the only pattern that emerges from standard synchronization. If we start to diverge from the original distribution given by O_3, O_4, O_5 , and O_6 , synchronization starts to disappear. On the other hand, if we use the criteria of quasi-synchronization, a

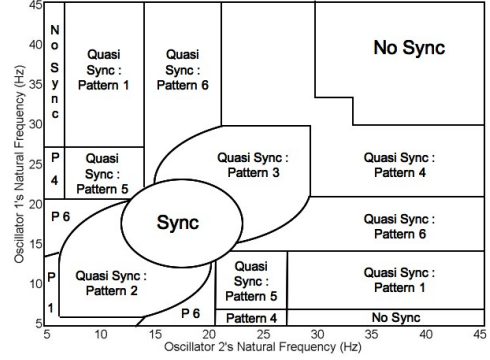


Figure 5. Synchronization regions emerging from a six-oscillator network response to varying frequencies of stimulus oscillators O_1 and O_2 . O_3, O_4, O_5 , and O_6 have couplings $k_{mn} = 1$, except $k_{12} = k_{21} = 0$, and have frequencies given by (3)-(4). Each numbered pattern corresponds to the quasi-synchronization of the following oscillators: (1) O_1 and O_2 ; (2) O_1, O_2 , and O_3 ; (3) O_2, O_3 , and O_4 ; (4) O_3 , and O_4 ; (5) O_1 with O_2 and O_3 with O_4 (but not O_1 with O_3 and so on); (6) O_2 and O_3 . The elliptical area around 17.5 Hz corresponds to all oscillators synchronized (as in Fig. 2), whereas the “No sync” areas correspond to no synchronization of oscillators (as in Fig. 1).

total of eight possible patterns emerge — patterns (1)–(6), plus all oscillators synchronized, plus no oscillators synchronized. We should compare this to the binary sync/no-sync possibilities when we use a more strict sense of synchronization.

It is often argued that neural synchronization may be used by the brain because it allows firing rates to reach above a certain response threshold. One possible criticism of Definition 1 is the lack of such feature. Though this may be true in a strict sense, if we look at the simulations shown in Fig. 3, the phases of oscillators lag shortly behind each other. Thus, if we think of oscillators as not being in the same place, time lag effects may yield similar effects.

Let us now see how we can use synchronization for pattern recognition. A specific stimulus may give rise to a specific synchronization pattern of the recognition oscillators. We will consider this pattern as the recognition of the stimulus by the network. In order to compute this recognition, we set:

- 1) Parameters: The stimuli, $\{(A_n^s, f_n^s, \phi_n^s(0))\}_{n \in G_S}$, the recognition oscillators' natural frequencies $\{f_n\}_{n \in G_R}$, and the coupling constants $\{k_{nm}\}_{n,m}$.
- 2) Initial Conditions: $\phi_n(0) = \phi_n^s(0)$ if $O_n \in G_S$, and randomly distributed in $[0, 2\pi]$ otherwise.
- 3) Dynamics: For $0 < t \leq T$, T constant, the phases follow Kuramoto's (1).

First, we address the model's robustness to noise. We start by assuming that the natural frequencies of exci-

tation of the sensory oscillators have a stochastic component. In other words, the natural frequency depends on time as $f_n(t) = f_n + \rho_n(t)$, where f_n is the original noise-free natural frequency of the stimulus oscillator ($O_n \in G_S$) and $\rho_n(t)$ is a zero-mean Gaussian white noise. A simulation for the network of six oscillators under noisy stimuli is graphed in Fig. 6. Figs. 6 and 7 show that synchronization is not much affected by the noise, so that the recognition of a stimulus seems to be robust under Gaussian noise. Starting from this observation, we conduct pattern recognition tests in noisy environments.

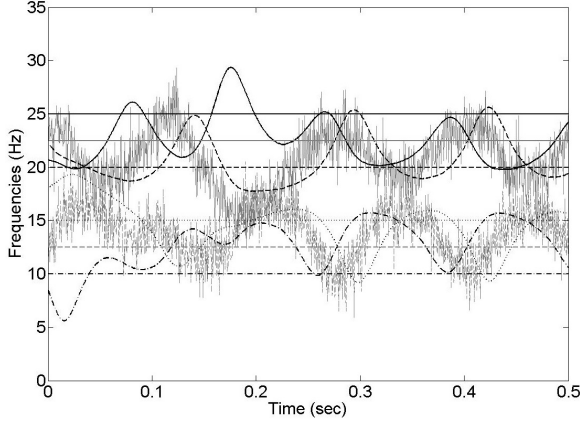


Figure 6. Six-oscillator network under noisy stimuli. The simulation parameters are the same as in Fig. 3, except that the frequencies of the stimulus oscillators are noisy, with f_1 and f_2 replaced by $f_1 + \rho_1(t)$ and $f_2 + \rho_2(t)$, and with $\rho_i, i = 1, 2$ being Gaussian white-noise distributions with mean zero and variance ten.

Given a randomly generated set of M stimuli, $s_1, s_2, \dots, s_M$, each composed of N_S natural frequencies, initial phases, and amplitudes, we use a set of noisy versions of these stimuli, $\{s_1^{(i)}, s_2^{(i)}, \dots, s_M^{(i)}\}_{i=1..Q}$, where Q is the number of samples, to obtain the recognition rate. Here we define the recognition rate as the rate of successfully recognized stimuli over all trials. The noisy stimuli are produced in the following way. For stimulus $s_j^{(i)}, i = 1, \dots, Q, j = 1, \dots, M$, there are N_S natural frequencies, $f_{j,r}^{(i)}, r = 1, \dots, N_S$. The time-varying stochastic natural frequencies of oscillator O_r in stimulus s_j and version i are given by $f_{j,r}^{(i)}(t) = f_{j,r}^{(i)} + \rho_{j,r}^{(i)}(t)$, where $f_{j,r}^{(i)}$ are normally distributed around $f_{j,r}$ of O_r in s_j (dealing with the slight differences between different occurrences of the same stimulus), and $\rho_{j,r}^{(i)}(t)$ is a zero-mean Gaussian white noise modeling the synaptic noise. To evaluate the ability of a network of oscillators to correctly recognize noisy stimuli, we made simulations according to the following three procedures.

- 1) Fix a time T , the number N_R of recognition oscillators, their natural frequencies, $f_1, f_2, \dots, f_{N_R}$,

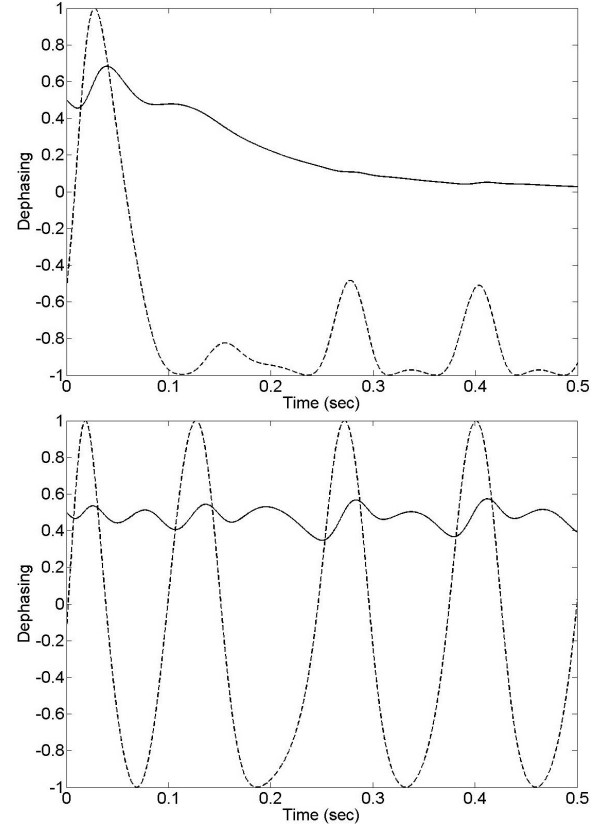


Figure 7. Variance of the sine of dephasing (solid line) between O_3 and O_4 (top) and between O_3 and O_5 (bottom) under the noisy stimulus shown in Fig. 6. The dashed line shows the phase difference between oscillators. According to Definition 2.1, O_3 and O_4 synchronize, whereas O_3 and O_5 do not.

and the set of symmetric coupling constants k_{nm} .

- 2) Compute a set of initial patterns, $P_1, P_2, \dots, P_M$, associated to the clean stimuli $s_1, s_2, \dots, s_M$, as described in Section III. The P_i 's can be thought of as a binary matrix with element one at the a -th and b -th lines if, at time T , oscillators O_a and O_b are ϵ -synchronized and zero if they are not.
- 3) Compute patterns associated to the noisy stimuli $\{s_1^{(i)}, s_2^{(i)}, \dots, s_M^{(i)}\}_{i=1..Q}$. For each of these patterns, find which $P_1, P_2, \dots, P_M$ is closest to it using a Hamming measure. If the closest initial pattern corresponds to the same stimulus without noise, recognition is successful, otherwise not. The percentage of successful recognitions is the rate of recognition.

It is instructive to relate the above steps to classical conditioning theory. We think of the stimuli as the unconditional stimuli, and the set of initial patterns $P_1, \dots, P_M$ as the unconditioned responses associated to the stimuli. Later on, we will see how we can include in the model conditioned stimuli that become associated to conditioned responses.

For our simulation, we chose $T = 500$ ms and $M = 5$. The natural frequencies of each stimuli were independently and uniformly drawn between 5 and 45 Hz, a range corresponding to observed frequencies in the brain's cognitive activity [35], [36], [33], [37], [32]. For simplicity, initial stimulus phases were set to zero and amplitudes to one. We considered sets of stimuli composed of 2, 5, and 10 frequencies. For the noise, $\rho_{j,r}^{(i)}(t)$ had standard deviation equal to 10, with a noise at the same order of magnitude as the natural frequencies. The coupling constants between recognition oscillators and between stimulus oscillators were uniformly distributed on the interval $[0, 0.002]$, but the coupling between stimulus and recognition oscillators should be stronger, and so were uniformly drawn from the interval $[0, 2]$. In Fig. 8, we show the recognition rates for different numbers of recognition oscillators and stimulus frequencies. We see that increasing the computational capacity of the recognition network, i.e. increasing the number of recognition oscillators, leads to an improvement on the recognition rate. The figure also shows that more complex stimuli, with a larger number of natural frequencies, have better recognition results. In fact, in the simulation it is a stronger effect than the number of recognition oscillators.

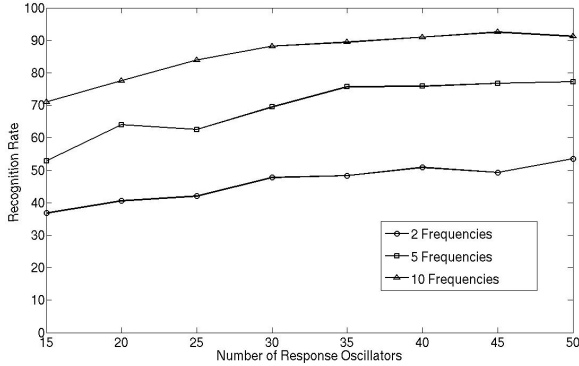


Figure 8. Recognition rates as a function of the number of recognition oscillators. Rates were averaged for different random values of frequencies, couplings, and noise, for a total of 125 trials.

In Fig. 9, we use 30 recognition oscillators and compute the average recognition rates when T , the oscillator-computation time, varies between 0.1 s and 1.0 s. Longer computation times allow oscillators to synchronize better in a noise-robust manner. Furthermore, the larger the number of stimulus oscillators, the lower the time needed for good recognition rates. Note that most of the gain from having a longer time to synchronize occurs in the first 500 ms.

In Fig. 10, we set $T = 0.5$ s and studied the recognition rates as a function of the mean value of the coupling constants between stimulus and recognition

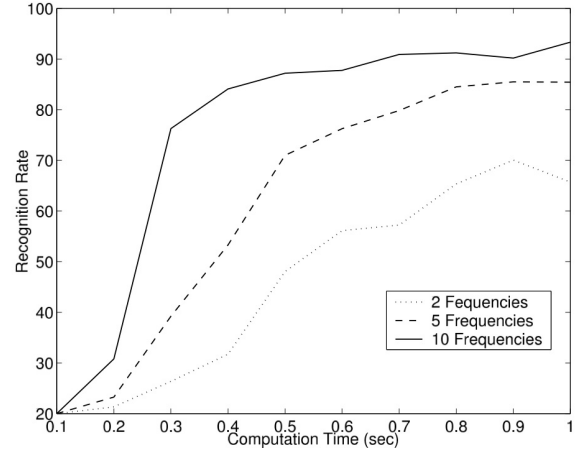


Figure 9. Recognition rates as a function of the time (averaged over 125 trials). The parameters are the same as before.

oscillators. This figure indicates the existence of an optimal value for these constants, nearly independent of the number of natural frequencies of the stimulus oscillators. Below this optimal value, the network is not sensitive enough to external stimulation; above it, any excitation will lead to a synchronization of all oscillators and no discrimination. This leads us to consider the coupling constants more as related to sensitivity parameters than to learning, contrary to the standard view of Hebb's rule for neural networks. In the next section, we discuss other mechanisms for learning that do not involve changes to coupling strengths.

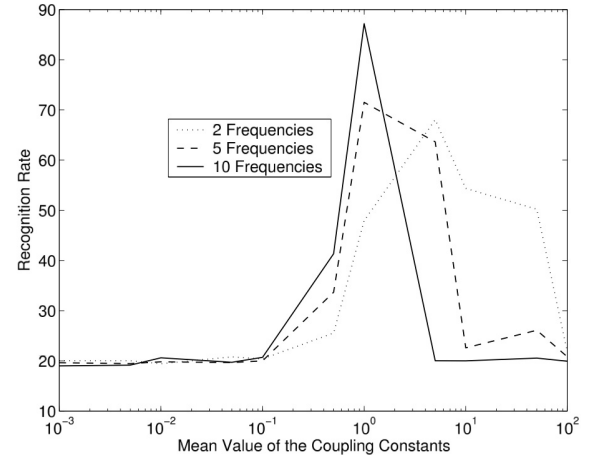


Figure 10. Recognition rates as a function of the mean value of the coupling constants' strength for 30 recognition oscillators (averaged over 125 trials).

Fig. 11 shows the variation of the recognitions rates as a function of the size of the set of stimuli. While an increase in the number of stimuli lowers the recognition rate, it is remarkable that 30 recognition oscillators are able to correctly recognize 10-frequency stimuli with

rates greater than 60% for a 40 stimuli set. To stress this point, we should recall that the rate of recognition by chance would be only 2.5%.

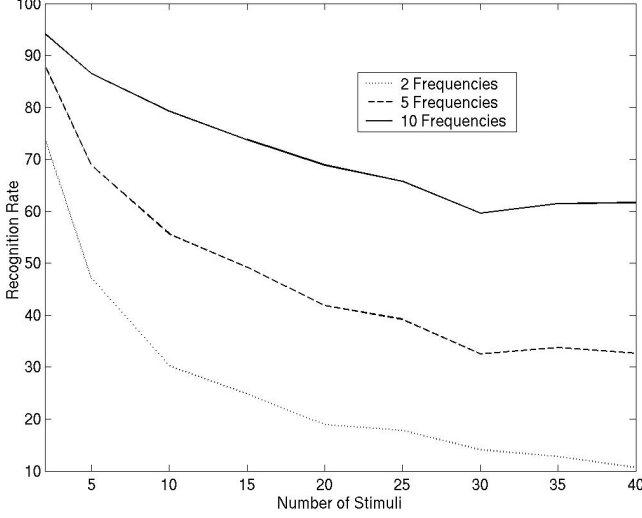


Figure 11. Recognition rates as a function of the number of stimuli in a network of 30 recognition oscillators (averaged over 125 trials). We use the same parameters as above.

III. LEARNING PATTERN RECOGNITION

While recognition of stimuli is in itself important, one of our main interests in this paper is to have a network that learns by reinforcement to associate a stimulus to a pattern [32]. In this section we introduce such learning. Since, as we discussed earlier, frequencies seem more important than couplings, in our model, memory is encoded on the recognition oscillators' frequencies. Reinforcement changes the recognition oscillators' natural frequencies. To model this change, during *reinforcement* we postulate the following dynamic for the *natural frequencies*, in addition to (1),

$$\frac{df_n(t)}{dt} = \sum_{O_m \in G_R} \mu_{nm} \left(\frac{1}{2\pi} \frac{d\phi_m(t)}{dt} - f_n(t) \right). \quad (7)$$

Equation (7) drives the natural frequency f_n to a value closer to the instantaneous frequency given by the dynamics. We emphasize that, because $f_n(t)$ is the natural frequency of the n -th oscillator, and not the time derivative of its phase, (7) is not a second-order differential equation. The coefficients $\{\mu_{nm}\}_{nm}$ ($\mu_{nm} = \mu_{mn}$) are learning parameters, chosen such that the system evolves towards the desired pattern. If O_n and O_m are to synchronize, we choose $\mu_{nm} > 0$; if they are not to synchronize, we set $\mu_{nm} < 0$; when it is indifferent whether they synchronize, we have $\mu_{nm} = 0$. To make it explicit whether a learning parameter is bringing

frequencies together ($\mu > 0$) or pushing them apart ($\mu < 0$), we call them μ_+ and μ_- , respectively. The procedure for learning may be summarized as follows.

- **Parameters:** The stimulus, $\{(A_n^s, f_n^s, \phi_n^s(0))\}_{n \in G_S}$, the recognition oscillators' natural frequencies $\{f_n\}_{n \in G_R}$, and the coupling constants $\{k_{nm}\}_{n,m}$, and the learning parameters $\{\mu_{nm}\}_{n,m}$.
- **Initial Conditions:** For $O_n \in G_S$ we set $\phi_n(0) = \phi_n^s(0)$ and $f_n(0) = f_n^s$, otherwise $\phi_n(0)$ is randomly distributed in $[0, 2\pi]$.
- **Dynamics:** For $0 < t \leq T$,

$$\frac{1}{2\pi} \frac{d\phi_n}{dt} = f_n + \rho_n(t) + \sum_{m=1}^N A_n A_m k_{nm} \sin[\phi_m(t) - \phi_n(t)] \quad (8)$$

For all $O_n \in G_R$,

$$\frac{df_n(t)}{dt} = \sum_{O_m \in G_R} \mu_{nm} \left(\frac{1}{2\pi} \frac{d\phi_m(t)}{dt} - f_n(t) \right) \quad (9)$$

Let us consider a fully connected three-node network, where O_1 is a stimulus oscillator and O_2 and O_3 are recognition oscillators. In this network, only two patterns can occur: either O_2 and O_3 are synchronized or not. We choose as initial values $f_2 = 10$ Hz and $f_3 = 35$ Hz. If a stimulus with frequency $f_1^s = 25$ Hz occurs, no synchronization emerges. Let us now assume that we would like O_2 and O_3 to learn to synchronize under this stimulus.

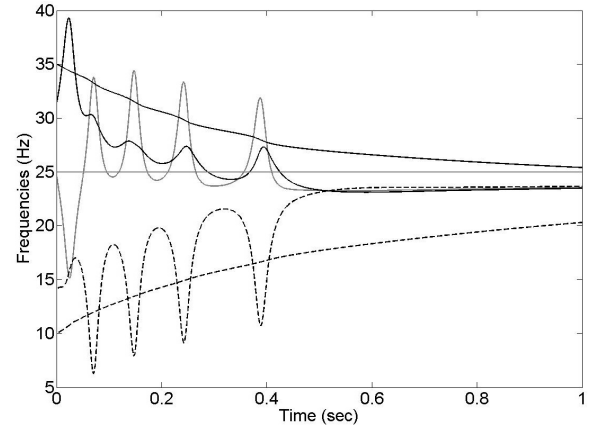


Figure 12. Learning in a three-oscillator network. Oscillators initially unsynchronized get synchronized when the coupling is positive ($\mu_{23} = 0.3$) due to changes in their frequencies. The instantaneous frequencies of O_1 , O_2 , and O_3 are shown as grey solid, black dotted, and black solid lines. The straight lines depict the oscillators' natural frequencies. The instantaneous frequencies are the lines that oscillate, while the natural frequencies slowly converge to their final values.

In Fig. 12, the recognition oscillators frequencies evolve towards the stimulus', eventually synchronizing. If we

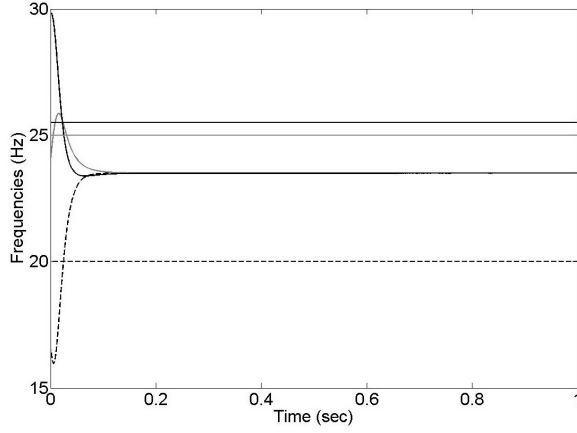


Figure 13. Oscillators' frequencies after learning in a three-oscillator network. The same representation as Fig. 12 is used for the oscillators' instantaneous and natural frequencies.

now use the new learned frequencies, shown in Fig. 13, the stimulus results in the synchronization of O_2 and O_3 . Finally, if we want the network to unsynchronize and forget, we can simply use a negative μ_{23} .

Let us now consider the more complicated case of the six-oscillator network studied earlier. Figs. 14–17 show the response pattern to a stimulus with frequencies $f_1^s = 8$ Hz and $f_2^s = 12$ Hz before and during learning. According to our criteria of ϵ -synchronization, O_3 is synchronized with O_4 but not with O_5 . For the network to learn to synchronize O_3 , O_4 , and O_5 , we set $\mu_{34} = \mu_{35} = \mu_{45} = 0.3$, $\mu_{n6} = 0$, $n \in \{3, 4, 5\}$ (with $\mu_{nm} = \mu_{mn}$).

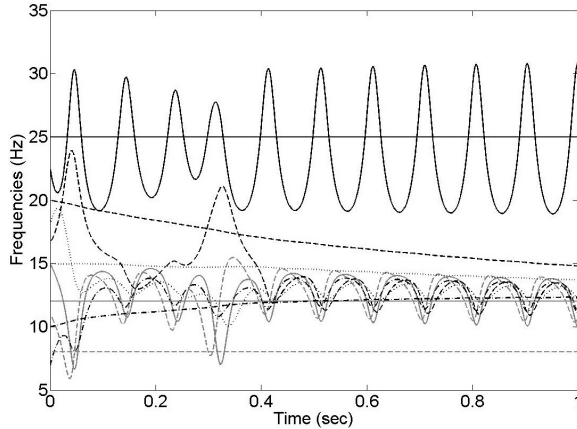


Figure 14. Instantaneous frequency for a six-oscillator network during learning. Parameters are given by (3) and (4), and the stimulus is the same as in Fig. 1, except for $\omega_1^s = 8$ Hz and $\omega_2^s = 12$ Hz. Oscillators $O_1, \dots, O_6$ are represented as in Fig. 1.

We now go back to the problem of stimuli recognition, and we show how adapting the natural frequencies of the recognition oscillators during reinforcement improves the recognition rates. We adopt a similar setup to the

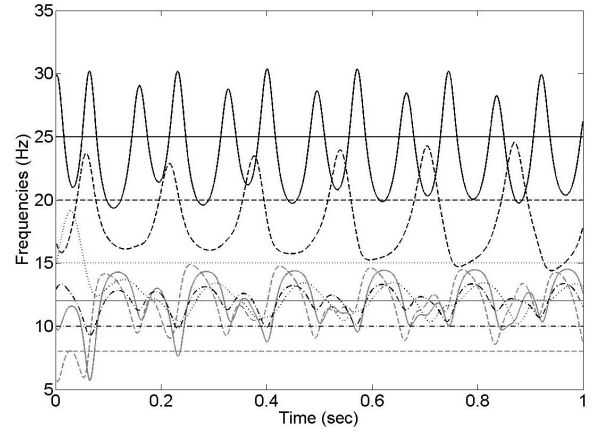


Figure 15. Instantaneous frequencies for the six-oscillator network of Fig. 14 after learning.

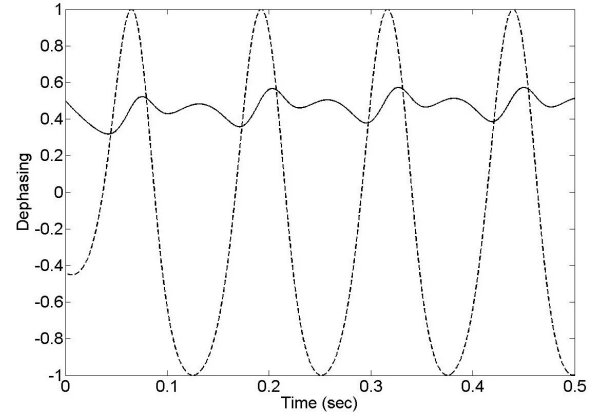


Figure 16. Variance of the sine of dephasing (solid line) and dephasing (dashed line) for the six-oscillator network in Fig. 14.

one used in Section III, with the main difference that only noisy versions of the stimuli are used and that some are considered reinforcement, based on the model described in [?, Chapter 8]. During reinforcement, the natural frequencies of the recognition oscillators evolve according to (8) and (9), and the patterns used for stimulus recognition are updated. More precisely, if we set the number L of noisy versions used for learning, we and go through the following steps.

- 1) Fix the oscillator simulation time T , the number of recognition oscillators N_R , their natural frequencies $f_1, f_2, \dots, f_{N_R}$, and the connection strengths k_{nm} .
- 2) Compute the initial synchronization patterns $P_1^1, P_2^1, \dots, P_M^1$ associated to the noisy stimuli $s_1^{(1)}, s_2^{(1)}, \dots, s_M^{(1)}$, as described in Section III.
- 3) For $i = 2, \dots, L$ and $l = 1, \dots, M$, compute the synchronization pattern $P_l^{(i)}$ associated to $s_l^{(i)}$. Then, use the learning parameters $\mu_{nm}^{il} = \mu_+^{(i)}$ if recognition oscillators m and n are synchronized

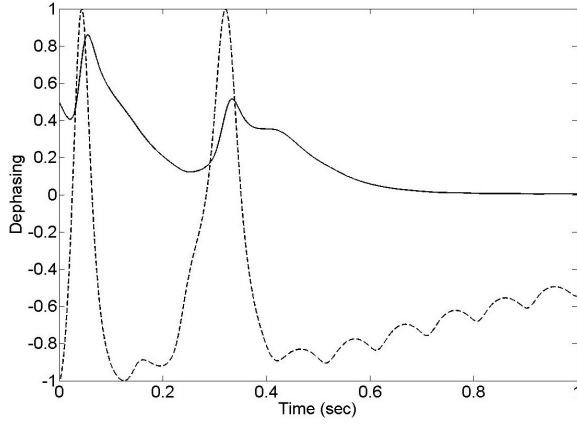


Figure 17. Variance of the sine of dephasing (solid line) and dephasing (dashed line) for the six-oscillator network in Fig. 15.

for $P_l^{(i-1)}$ but not for $P_l^{(i)}$, $\mu_{nm}^{il} = \mu_-^{(i)}$ if they are not synchronized, and $\mu_{nm} = 0$ otherwise (μ_+ and μ_- corresponding to positive and negative values). Evolve, according to (8) and (9), the natural frequencies of the recognition oscillators. The pattern at the end of this reinforcement, P_l^i , is the updated recognition of stimulus s_l .

- 4) For $i = L + 1, \dots, Q$, compute the patterns associated to $s_1^{(i)}, s_2^{(i)}, \dots, s_M^{(i)}$, and find which of the updated representation patterns $P_1^L, P_2^L, \dots, P_M^L$ is the closest to it according to a Hamming measure. If the closest initial pattern corresponds to the correct stimulus, then the recognition is correct.

We applied steps 1–4 to various sequences of learning parameters μ_{nm} . For simplicity, we considered only cases where $\mu_{mn} = \mu_+$ when m and n were to synchronize and $\mu_{mn} = \mu_-$ otherwise. We also fixed $T = 500$ ms, the number of stimulus oscillators to 5, and the number of recognition oscillators to 30. We defined the rate of recognition as the percentage of correctly recognizing a noisy stimulus to the original one. Noise was the same as before. The first relevant general result we obtained was that the same value for the μ parameters at each trial does not result in an improvement of the recognition rates. Indeed, either the parameters are low enough so that (7) would be negligible and learning would not occur, or they are high enough such that (7) is not to be negligible. If the latter, frequencies of the recognition oscillators evolve back and forth, without any convergence. In this case, the patterns for the stimuli at a given step do not match the patterns obtained at the next step, and the recognition rate drops down to random guess rates. However, using a decreasing sequence of the learning parameters, the patterns converge, followed by a noticeable improvement on the recognition rates. We also noticed that μ_- must be significantly smaller than

μ_+ for learning to happen. Fig. 18 shows (solid black line) an example with the parameters set as $\mu_+^{(1)} = 7.3$, $\mu_+^{(i+1)} = \mu_+^{(i)} / (i + 1)$, and $\mu_-^{(i)} = -\mu_+^{(i)} / 2$, where i is the trial number. One interesting characteristic of the learning shown is the initial period when the oscillators frequencies are adapting very fast. This fast adaptation leads to an initial mismatch between representations and new patterns, and to a dip in the recognition rate, before the rates finally improve by 15%. We emphasize that, because of the dynamics of the model, this dip in recognition rate will necessarily occur. Furthermore, starting values for μ equal to those used after the fifth reinforcement, when better rates appear, has no effect, since these coefficients are too small. Fig. 18 also shows

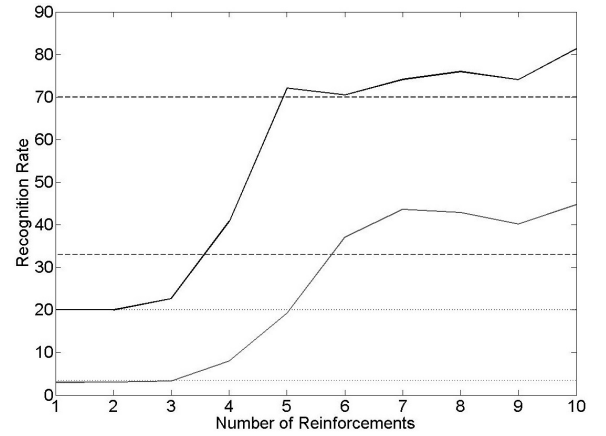


Figure 18. Recognition rate as a function of the number of reinforcement trials for 5 (black line) and 30 (grey line) distinct stimuli.

a similar computation for a larger set of 30 stimuli (solid grey line). Smaller values, by a factor 2, for μ led to better learning, implying that learning more stimuli must take more reinforcement trials. Recognition rates after ten reinforcements improved approximately 12%, which constitutes a 37 % improvement over the initial 33 % before learning. One interesting aspect of our model is that the mean rate of learning, shown in Fig. 18. Although standard stimulus-response theories do not present the observed dip in recognition observed in Fig. 18, this type of behavior resembles those of interference in psychology. The basic idea is that past learning can interfere with learning a new related concept or behavior that has serious overlap with the old. Suppes [31] studied a case where children at about the age of five years old can learn rather easily when two finite sets are identical, but this learning interferes with learning the concept of two sets being equivalent. A neural network that models this result is given in [36], and the results presented therein are quite similar to the rates in Fig. 18.

As a last topic in this section, we focus on storage capacity. In Fig. 5, we showed that by adopting the concept of quasi-synchronization, we were able to have eight different patterns stored in the network, as opposed to just two with the standard definition of synchronization. Additionally, our 5+30 oscillator network above was able to recognize 30 different *noisy* stimuli with a rate of almost 50% (see Fig. 18). So, our simulations suggest that our model with N oscillators has a storage capacity proportional to N . This should be contrasted with the storage capacity of Hopfield networks, a well-known model of artificial neural networks. In his famous 1982 paper, John Hopfield showed that a simple integrate and fire neural network could be used as an associative memory [11]. Hopfield determined that the storage capacity of his network, given in terms of different patterns that could be recovered, was $0.15N$, where N is the number of neurons. Later on, McEliece and collaborators [21] showed in more detail that the limiting storage for a Hopfield network was $N/(2 \log N)$, which corresponds to approximately five patterns for a network of 35 neurons. We thus see that, by using oscillators in the way proposed in this paper, we achieve a storage capacity that seems significantly larger than that of Hopfield networks.

IV. APPLICATION TO IMAGE RECOGNITION

We now show an example of how we can use quasi-synchronized networks of oscillators to recognize images. In this example we investigate two cases of interest: (i) recognition of images degraded by Gaussian noise; (ii) recognition of incomplete images. The patterns used in our image recognition example are shown in Fig. 19. We start with the one-step learning performance of a 30-

0 1 2

Figure 19. Digitized version of the numbers 0, 1, and 2, with resolution 80×107 . Each number is represented by 8560 black-and-white pixels.

oscillator network. In our analysis, we use the following procedures.

- 1) Center the pictures and represent them as a single sequence of 0's and 1's corresponding to the 8560 pixels.
- 2) Compute the discrete time Fourier transform of the sequence found above.
- 3) Select the ten largest Fourier coefficients for each picture.
- 4) Define the noise-free stimulus for the digitized picture of 0 as $s_{zero} = \{(A_n^{zero}, f_n^{zero}, \phi_n^{zero}(0))\}_{1 \leq n \leq 10}$, where A_n

and ϕ_n are the amplitude and phase of the coefficients obtained on step 3.

- 5) Repeat step 4 for stimuli 1 and 2.

The stimuli s_{zero} , s_{one} , and s_{two} obtained from 1–5 are noise-free. To obtain the noisy version of the stimuli, for example, we just inject noise into the stimuli and repeat 1–5.

For (i), we simulate the influence of Gaussian noise on all pixels. The recognition rate of the noisy version as a function of the signal to noise ratio (SNR) is depicted on Fig. 20. To compute these rates, we drew

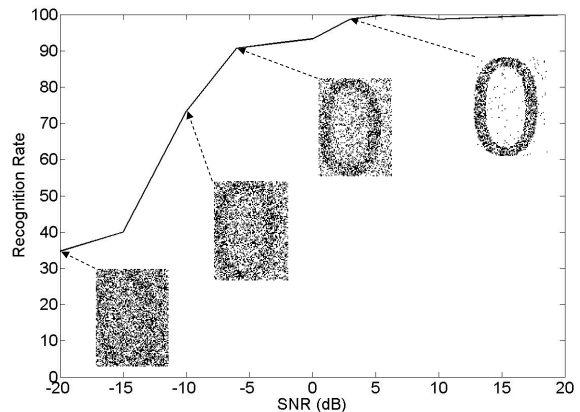


Figure 20. Correct recognition rates of the characters 0, 1, and 2 by a network of 30 quasi-synchronized oscillators as a function of the signal-to-noise ratio (SNR) of a Gaussian noise. The figure shows examples of different noise levels for the patterns. We can see that for -10 dB the pattern is barely visible, yet the network correctly recognizes the pattern more than 70% of the time.

various networks at random (response oscillators natural frequency and coupling constants), and to each network we presented the noise-free version of stimuli zero, one, and two. Then we presented 5 noisy versions of the same stimuli. Whenever the synchronization pattern occurring with the noisy version of one of the three stimuli is closer (Hamming distance) to the non noisy version of the same stimuli than the two other, we declared a successful recognition. The percentage of successes with respect to the number of trials is the recognition rate. Then we averaged the rates of all the networks drawn at random.

For (ii), we studied the influence of a "hole" in the picture. White squares of various size were superposed to the pictures on a random position (one white square per noisy picture). The same process as (i) was applied to obtain the recognition rates. Fig. IV shows the decrease of the recognition rates as a function of the white square size for two situations: without any Gaussian noise, and with a SNR of -6dB. We see that holes are more harmful to recognition rates than Gaussian noise. When the hole is of big size (40×40 pixels), the impact of Gaussian noise seems to become negligible.

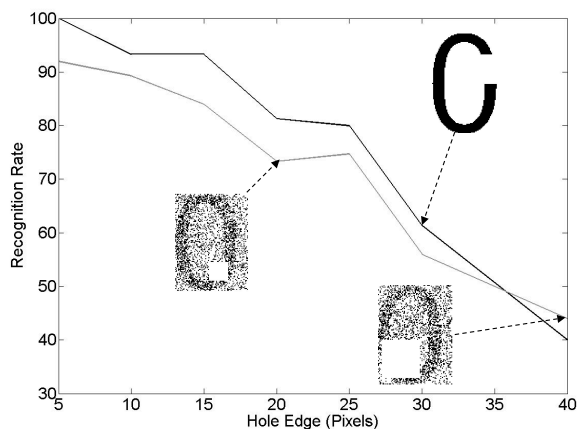


Figure 21. Correct recognition rates of the characters 0, 1, and 2 by a network of 30 quasi-synchronized oscillators as a function of the size of a blank square hole randomly positioned on the picture. The dark line shows the rates for the noise-free pictures, whereas the gray line shows it pictures with a SNR of -6 dB. The figure shows three examples of the picture “0” for different values of the hole size and noise.

Secondly we study the influence of learning with the reinforcement process described in the previous section. We test various hole sizes with a Gaussian noise so that the signal to noise ratio is -6 dB. To compute the recognition rates we draw networks at random and start by computing the synchronization pattern with a *noisy* version of the three stimuli. Then 10 other noisy versions of the three stimuli are used for learning reinforcement purposes. Finally, 5 other noisy versions are used to test the recognition capability of the networks that went through the learning procedure. The recognition rate are compared to mean one step learning rates initialized with all the noisy versions used for learning. As shown on Fig. 22, learning yields an improvement of up to 14%

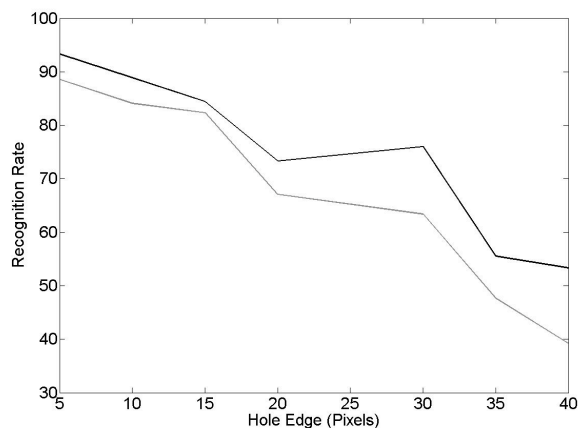


Figure 22. Recognition rates for different holes, before (gray line) and after learning (black line). The largest rate improvement was for the case when the rate jumped from 39% (almost at chance level) to 53% after learning (well beyond chance level).

on the recognition rates. Compared to the rate of 39%

for holes of edge size 40, this increase in the recognition rates represents a 36% improvement of the performance. We also emphasize that learning is more efficient when recognition rates are initially lower (without learning).

V. FINAL REMARKS

The three main features of this article have all been described, but we summarize them here to bring out what is most significant. First, we use a stochastic and approximate definition of synchronization suitable for the noisy environment found in many biological applications, where noise is endemic and cannot easily be removed. Second, our model simulations demonstrated robustness in the presence of noise, again a necessary feature for most biological applications. Third, and finally, we showed how a network of oscillators can learn to recognize a set of patterns with noise by changing their natural frequencies, rather than changing their coupling strengths, as in Hebb’s rule.

Given its importance in artificial neural networks, it is worth comparing some of our results with those obtained for Hopfield networks [11]. First, we saw that by using quasi-synchronous oscillators, we were able to recognize a much larger number of patterns than one would if we were to use Hopfield nets. In fact, the computed theoretical limit for a 35-node Hopfield network is approximately five patterns [21], but quasi-synchronized oscillators were able to recognize thirty. This indicates a higher storage capacity than that of Hopfield networks. Another important distinction between our model and Hopfield’s is in the way we represent learning. In our model, the oscillators’ couplings are fixed, but their natural frequencies vary. In Hopfield’s model, learning happens by changes in the connections between each node. Because it is fully connected, it is very hard to produce computer chips that mimic large-scale Hopfield network [23]. It is possible that by having fixed connections but changing natural frequencies, our model may face less difficulties with hardware implementation.

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