

Beyond the quantum formalism: consequences of a neural-oscillator model to quantum cognition

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Abstract In this paper we present a neural oscillator model of stimulus response theory that exhibits quantum-like behavior. We then show that without adding any additional assumptions, a quantum model constructed to fit observable pairwise correlations has no predictive power over the unknown triple moment, obtainable through the activation of multiple oscillators. We compare this with the results obtained in reference [5], where a criteria of rationality gives optimal ranges for the triple moment.

Introduction

Recently, much attention has been paid to quantum-mechanical formalisms applied to human cognition (see [6, 8, 9], and references therein). This comes from an increasing set of empirical data better described by quantum models than classical probabilistic ones (for an new effective classical approach, however, see [7], to appear in this proceedings).

The underlying origins of such quantum-like features are not well understood, but few researchers believe that actual quantum mechanical processes are responsible (see [6] but also [10] for a different view). Instead, as argued in [4], what is behind such features is a contextual influence. Interference-like effects in neuronal firings in the brain lead to outcomes that are context dependent, similar to the two-slit experiment in quantum mechanics, thus providing a possible explanation. In fact, in [3] we showed how a simplified neural model with interference emerging from the collective dynamics of coupled neurons gives origin to quantum-like effects. Such model was designed to be consistent with currently known neurophysiology and to reproduce the behavioral stimulus-response theory [11]. Here, we discuss the implications of

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such neural model to quantum cognition, and in particular to the predictability power of the quantum-mechanical apparatus, as opposed to its descriptive power.

Model and main results

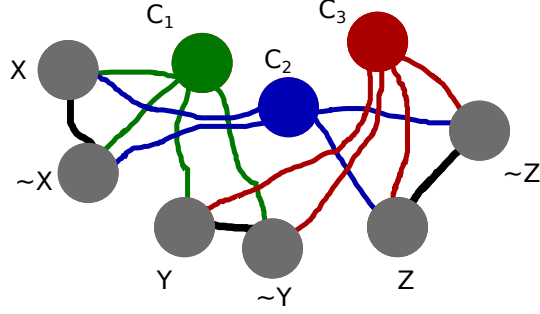
Here we briefly present the main model shown in [3, 11], and the readers are referred to them for details. For the simple case of a continuum of responses, we start with representations of stimulus and response in terms of phase oscillators. Such oscillators, made out of collections of neurons, are synaptically coupled, and, depending on the coupling strength, may synchronize. Let $s(t)$ be the neural oscillator representing the activation of a stimulus, and $r_1(t)$ and $r_2(t)$ the oscillators for the two extremes in a continuum of responses. We focus on their phases, φ_s , φ_{r_1} , and φ_{r_2} , whose dynamics are given by

$$\dot{\varphi}_i = \omega_i + \sum_{j \neq i} k_{i,j}^E \sin(\varphi_i - \varphi_j) + \sum_{j \neq i} k_{i,j}^I \cos(\varphi_i - \varphi_j), \quad (1)$$

where $k_{i,j}^E$ and $k_{i,j}^I$ are the overall excitatory and inhibitory couplings between the neural oscillators. During reinforcement, the coupling strengths $k_{i,j}^E$ and $k_{i,j}^I$ are changed in a Hebb-like fashion. This model can easily be extended to include multiple stimulus and response oscillators. For instance, in [3] it was used with two stimulus oscillators to obtain quantum-like effects. Such effects were the consequence of couplings between the oscillators that were reinforced to respond to two different stimuli corresponding to incompatible contexts. When both stimuli were simultaneously activated, an interference effect was obtained.

Quantum-like models lead to contextual responses, in the sense that there exists no joint probability distribution for the associated random variables. Let us look at the particular example presented in reference [2] and expanded in another context in [5]. Let $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ be ± 1 -valued random variables, and consider the neural oscillator system represented in Figure 1. For this system, the activation of one of the three stimulus oscillators, C_1 , C_2 , or C_3 , leads to the corresponding responses computed via phase differences. For example, if C_1 is sampled, the oscillators' dynamics, dictated by the specific values of inhibitory and excitatory couplings, converge to a fixed point that may favor $\mathbf{X} = 1$ (oscillator X) instead of $\mathbf{X} = -1$ (oscillator $\sim X$), while at the same time favoring $\mathbf{Y} = -1$, thus corresponding to a negative correlation. With such oscillator system, it is possible in principle to choose couplings such that the correlations between $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ are too strong for a joint probability distribution to exist. As a consequence, and because of the pairwise commutativity of the set of quantum-mechanical observables $\hat{X}$, $\hat{Y}$, and $\hat{Z}$ corresponding to the random variables $\mathbf{X}$, $\mathbf{Y}$, and

Fig. 1 Layout of a neural-oscillator system exhibiting pairwise correlations between $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$. In this oscillator system, $(\mathbf{X} = 1) \& (\mathbf{Y} = -1)$ corresponds to the synchronization with oscillator C_1 closer in phase to X and not to $\sim X$, while at the same time being closer to Y than to $\sim Y$.



$\mathbf{Z}$, it follows that there exists no state $|\psi\rangle$ in the Hilbert space $\mathcal{H}$ where such observables are defined and such that the neural correlations hold [5]. However, even in such situations a quantum model can be constructed [1], and in order to describe the correlations set by the neural-oscillator model, we are forced to expand the Hilbert space to $\mathcal{H}' \otimes \mathcal{H}$ [5]. For instance, we can write a state vector

$$|\psi\rangle = c_{xy}|A\rangle|\psi_{xy}\rangle + c_{xz}|B\rangle|\psi_{xz}\rangle + c_{yz}|C\rangle|\psi_{yz}\rangle, \quad (2)$$

where $|A\rangle$, $|B\rangle$, and $|C\rangle$ are orthonormal vectors in $\mathcal{H}'$, $\langle\psi_{xy}|\hat{X}\hat{Y}|\psi_{xy}\rangle = -2/3$, $\langle\psi_{xz}|\hat{X}\hat{Z}|\psi_{xz}\rangle = -1/2$, $\langle\psi_{yz}|\hat{Y}\hat{Z}|\psi_{yz}\rangle = 0$, and c_{xy} , c_{xz} , and c_{yz} are such that $|c_{xy}|^2 + |c_{xz}|^2 + |c_{yz}|^2 = 1$. Because each of the states $|\psi_{xy}\rangle$, $|\psi_{xz}\rangle$, and $|\psi_{yz}\rangle$ can have arbitrary triple moments (they do not fix enough of the distribution) between -1 and 1 , it follows that (2) can describe the correlations but has no predictive power with respect to the neural oscillator model or human decision making.

However, the couplings encoding different responses in the Kuramoto equations do determine, within a certain range, values for the triple moment. The triple moment would be the equivalent, following [3], of a simultaneous activation of all stimulus oscillators. Thus, the neural model would provide a definite prediction, in contrast with the quantum one.

Final remarks

Quantum formalisms applied to human cognition have shown a great potential for certain applications in the social sciences. However, one must ask how this is so, and also how predictive they are. For instance, as showed above, it is possible to devise a neural system whose quantum description has no predictive power. Thus, we could in principle design an experiment to test this neural system, but not its corresponding quantum description.

Could be some principle to be added to the quantum description that could provide predictions for outcomes of the experiment proposed? For example, in [5] we proposed a minimization principle as a normative decision for quantum-like inconsistencies, which allowed signed probabilities to move from a descriptive to a normative theory. Perhaps a principle of this type added to the quantum formalism could be not only normative but predictive as well. However, if we think that the underlying dynamics for quantum cognition is actually from the complex and contextual interaction of neurons, perhaps some similar principle from it should be added to the quantum description.

Finally, we would like to emphasize that the quantum approach has suggested interesting experiments in psychology. As such, it is a promising field not only because of its ability to describe experiments, but also for the intuitions it provides for thinking about context-rich situations. Therefore, understanding its limitations and perhaps extending it would be desirable.

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